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Application of structural bending differential equation algorithm in bracing composite structure

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Abstract. The purpose of this paper is to study the effect of bending differential equation algorithm on composite bracing structures. By analyzing the existing landslide composite supporting structure, a new type of diagonal bracing composite supporting structure is purposed. The diagonal bracing composite supporting structure changes the original cantilever bending structure into diagonal loads transfer structure. It reduces the load on the structure which under the huge thrust of landslide, and it has good mechanical properties. Based on the Fredholm–Volterra line load integral and displacement control equation, diagonal bracing composite supporting structure control condition is put forward, and the continuous structure algorithm based on the deflection equation is deduced. The analytical solution of diagonal bracing composite supporting structure the displacement and internal force are solved. Continuous structure algorithm based on deflection equation of brace composite supporting structure engineering design is optimized. The experimental results verify the flexural differential equation algorithm of continuous structure by the three-dimensional finite element analysis. Based on the above finding, it is concluded that the design of diagonal bracing composite supporting structure can be optimized by the flexural differential equation algorithm.

Key words. Flexural differential equation, continuous structure algorithm, diagonal bracing composite supporting structure, mechanics characteristic.

1. Introduction

Existing large-scale landslide treatment of timbering form is numerous at present, structure forms are also various, the main application structure forms are cantilever anti-slide pile, double row piles, etc. Cantilever anti-slide pile was applied at the earliest, related research is more thorough. But Cantilever anti-slide pile application depth is limited by its mechanical characteristics, and control deformation ability of

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Cantilever anti-slide pile is poor. These characteristics of the cantilever anti-slide pile cannot satisfy the requirement of landslide with large thrust. Double row pile as supporting form more research in recent years, double row pile research has achieved some results in domestic and overseas [1]. Based on the theory of plastic deformation and plastic flow theory, the pile lateral soil pressure formula is derived. The design idea and design method of the single pile and Multi-pile are purposed [2]. On the basis of predecessors' research, some researchers test the pile groups model under the action of the displacement of soil in 1997, and test results with boundary element program computing results have good consistency [3]. The model of soil was used to study the relative displacement of piles and soil, pile spacing and pile arrangement of its influence on soil arching between piles [4]. The calculation and stress distribution of double-row anti-slide pile in three kinds of typical landslide is calculated [5]. The comparative analysis of prestressed concrete anchor-stabilizing double–row piles and rigid frame anti-slide pile is purposed [6]. Double row pile theory research has made some achievements, but still failed to in giant, large rock landslide to obtain the very good application. In southwest China, many types of geological disasters and landslide, multistage complex landslide along with the urban expansion and construction appear constantly. From the characteristics of the giant, large damage rock landslides, this paper present a new diagonal bracing composite structure base on the research of existing timbering structure. diagonal bracing composite structure mainly aimed at a large thrust (generally more than 2000 kN/m) rock landslides. Diagonal bracing composite structure calculation method is: Improved the deflection control equation under the condition of different boundary control, makes it can get the optimal transfer ratio of diagonal bracing composite structure (the ratio of landslide thrust which transmit by diagonal bracing composite structure into rock mass and the total thrust). The method also can solve the internal force and displacement of diagonal bracing composite structure.

Diagonal bracing composite structure is mainly composed of two parts, the vertical bearing structure and diagonal bearing structure. Vertical bearing structure differs from the anti-slide pile and double-row piles structure and so on, its main function is to transfer thrust from the rock free face to diagonal bearing structure; Diagonal bearing structure is transfer the load which from the vertical bearing structure through the column brace to support structure and finally pass it to the lower rock mass [7]. The characteristics of diagonal bracing composite structure model is: Change the normal original cantilevered pile mechanic (the normal original cantilevered pile depends on the shear strength of section and bending strength of pile to resist the load) into a transferring patterns which transfer landslide thrust is primary, pile resistance is accessory [8].

2. Materials and methods

Diagonal bracing composite structure can be divided into four sections from rigid joint and fixed point [9]. The internal force and displacement of each section can be obtained by the deflection differential equation under the control condition (bending moment and shear is zero at the node which connect the vertical brace and diagonal bracing).

Using elastic foundation beam differential equation solve the internal force and displacement of vertical brace anchoring range, and using the displacement as the control parameter to unify the four-section deflection differential equation, end up with solving the simultaneous equations of the whole structure, finally be solving the simultaneous equations of the whole structure [10].

Assumption of continuous structure flexural differential equation algorithm:

Vertical brace of composite structure pile under the rectangular uniform load.

The cross-section width of vertical brace and diagonal bracing in composite structure is the same.

Vertical brace and diagonal bracing intersection node O is assumed a rigid node.

Rock mass of fixed section in vertical brace and diagonal bracing according to the elastic material consideration.

Each section of the block division in the table.

General solution of differential equation for each section can be solved, particular solution of differential equation should also be solving at the same time. Each section should be four initial conditions to completely solve the equation [11]. The displacement, rotation, bending moment and shear force of each section as the initial condition into the equation, solution of equations can be obtained.

Vertical brace cantilever section (OA) flexural differential control equation is

$$EI\frac{\mathrm{d}^4x}{\mathrm{d}y^4} = q\,, \tag{1}$$

where q means distributed load, E is the elastic modulus and I is the moment of inertia. Finally, x is the displacement.

The displacement, rotation, bending moment and shear displacement equation

$$x(y) = x_0 + \phi_0 y + \frac{M_0 y^2}{2EI} + \frac{Q_0 y_3}{6EI} + \frac{qy^4}{24EI}\,. \tag{2}$$

Here, x_0 means the initial displacement, ϕ_0 means the initial rotation, M_0 means the initial bending moment and Q_0 means initial shear force. Generally, ϕ denotes the rotation function, given by the prescription

$$\phi = \phi_0 + \frac{M_0 y}{2EI} + \frac{Q_0 y_2}{6EI} + \frac{qy^3}{24EI}\,.$$

Finally, β, which is the vertical brace pile deformation coefficient, is given as

$$\beta = \left(\frac{KB_\mathrm{p}}{4EI}\right)^{\frac{1}{4}} \tag{3}$$

3. Calculation and analysis for diagonal bracing composite structure

3.1. Diagonal bracing composite structure in the original design and optimization design comparative analysis

Comparative analysis between the flexural differential equation algorithm of continuous structure and design calculation method of actual landslide control project can optimize the design of practical project engineering. A certain slope project of Wulong county in Chongqing City adopted in 3 different form diagonal bracing composite structure. Based on type A structure as an example. In this paper, h_1 means the distance from intersection (Point A) of vertical bracing pile with diagonal bracing pile to the top of vertical bracing pile; h_2 means the distance from intersection (Point A) of vertical bracing pile with diagonal bracing pile to the fixed point of vertical bracing pile; h_3 means the distance of fixed section of vertical bracing pile. Symbol θ means the angle of vertical bracing pile from the horizontal plane. The flexural differential equation for optimized structure internal force compared with the original design size and internal force (see Fig. 1).

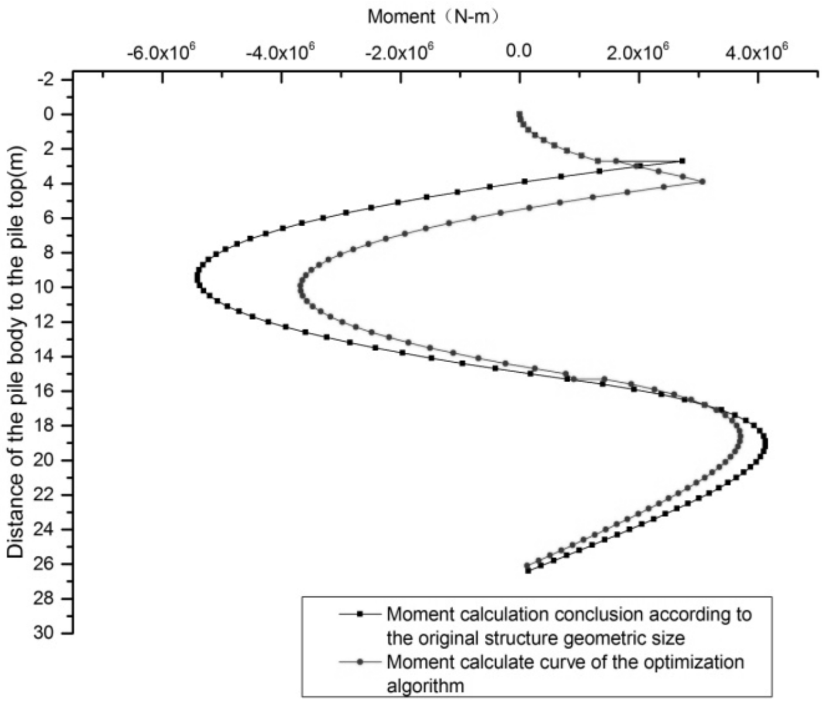

Fig. 1. Bending moment diagram of optimized structure and original design structure

From the figure, Point A to point B section of structure optimization design compared to the original design, the maximum bending moment increases 1727.3 kN.m (the original design maximum bending moment is 5408.4 kN.m, optimization design maximum bending moment is 3681.1 kN.m), optimization of structure relative to the original structure to reduce bending moment value is about 31.9 %. Point B to point C section of structure optimization design compared to the original design, the maximum bending moment increases 1019.8 kN.m (the original design maximum bending moment is 4124.2 kN.m, optimization design maximum bending moment is 3104.4 kN.m), optimization of structure relative to the original structure to reduce bending moment value is about 24.7 %.

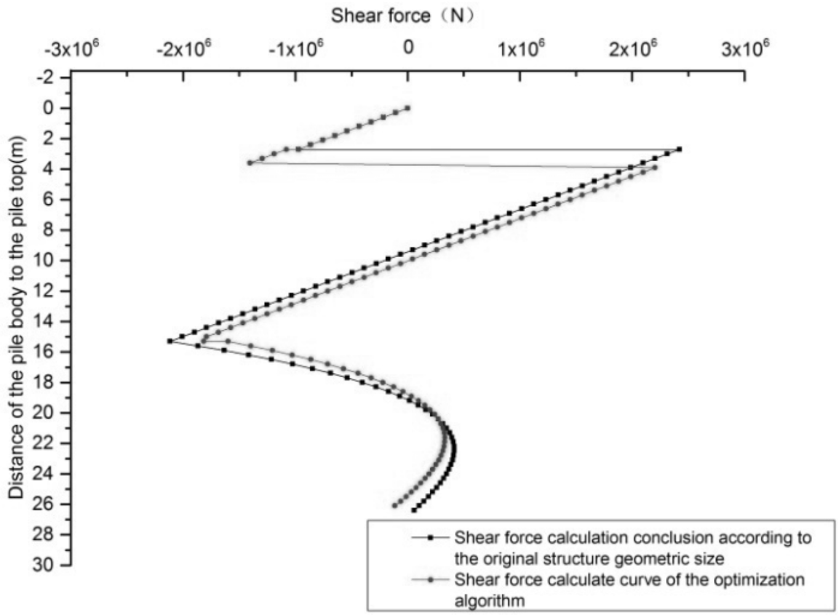

Fig. 2. Shear force diagram of optimized structure and original design structure

Figure 2 shows the shear force diagrams of optimized structure and original design structure. Point A to point B section of structure optimization design compared to the original design, the maximum shear increases 217 kN (the original design maximum shear is 2421 kN, optimization design maximum shear is 2204 kN), optimization of structure relative to the original structure to reduce shear value is about 8.9 %. Point B to point C section of structure optimization design compared to the original design, the maximum shear increases 296.5 kN (the original design maximum shear is 2114.7 kN, optimization design maximum shear is 1818.2 kN), optimization of structure relative to the original structure to reduce shear value is about 14.1 %.

Finally, Fig. 3 depicts the displacement diagrams of optimized structure and original design structure. In the original structural design of cantilever segments (point A to point O section), displacement variation is larger, the maximal displacement of the two methods difference in 1.2 mm; (increased from the two methods is 2.7 mm to 3.9 mm), in additional optimization design in the intersection (pointA) place does not appear A turning point. Two methods of comparison that the displacement of vertical bracing structure calculation difference of 1.2 mm, optimization of structure relative to the original structure to reduce bending moment value is about 44.4 %.

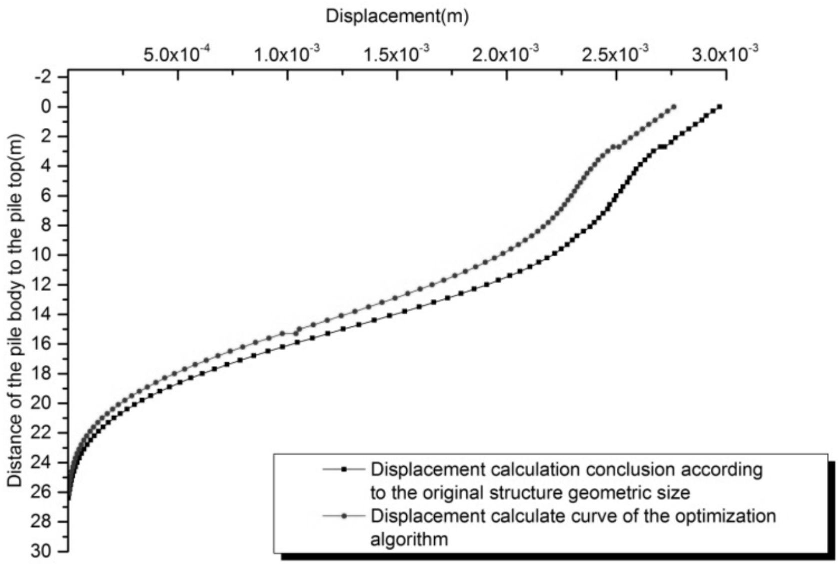

Fig. 3. Displacement diagram of optimized structure and original design structure

3.2. Diagonal bracing composite supporting structure and conventional supporting structure comparative analysis

For further verify the effect of diagonal bracing composite supporting structure. The internal force of anti-slide pile, reinforced anchorage pile, the original design and optimization design of diagonal bracing composite supporting structure were compared. Figure 4 shows the bending moment diagrams of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure.

Through the comparative analysis, bending moment value of the diagonal bracing composite supporting structure both in the original design and optimization design is much smaller than the reinforced anchorage pile and anti-slide pile. The main cause of the different between above supporting structure is mechanism different. Maximum bending moment value from large to small is: anti-slide pile $>$ anchor tensile pile $>$ the original design of diagonal bracing composite supporting structure $>$ optimization design of diagonal bracing composite supporting structure. Anti-slide pile structure because only rely on their own section strength (bending stiffness EI, tensile stiffness EA and shear stiffness GA) resistance to slide thrust, and therefore in fixed point bending moment value is maximum ($M_{\mathrm{Max}} = 61.5 \times 10^3$ kN.m).

Reinforced anchorage pile structure bending moment is smaller than anti-slide pile under the effect of pre-stressed anchor cable tension. Maximum bending moment is $M_{\mathrm{Max}} = 28.4 \times 10^3$ kN.m. Diagonal bracing composite supporting structure because of the difference of dynamic mechanism, a maximum bending moment are

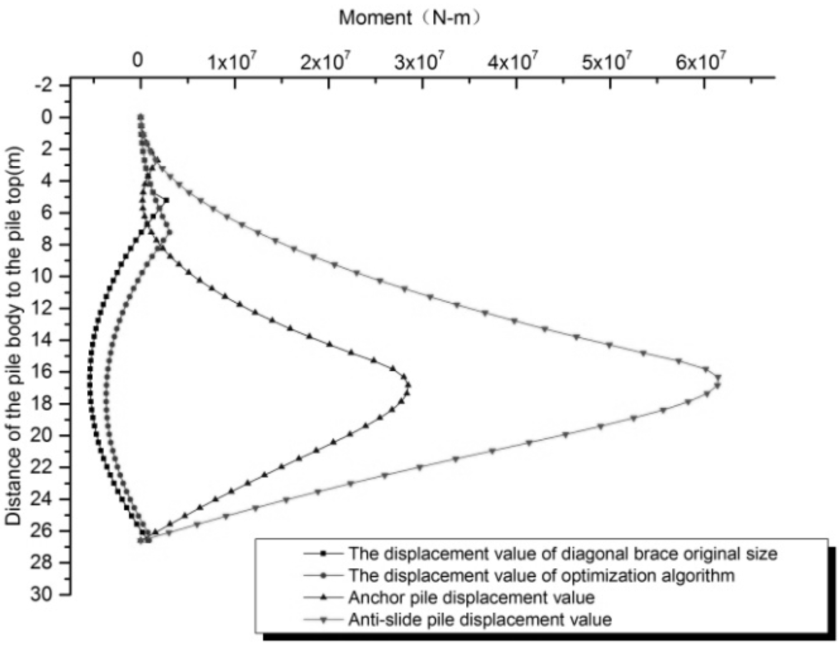

Fig. 4. Bending moment diagram of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure

much smaller than the front two structures. Maximum bending moment of original design diagonal bracing composite supporting structure is $M_{\text{Max}} = 5.41 \times 10^3$ kN.m. Maximum bending moment of optimization design diagonal bracing composite supporting structure is $M_{\text{Max}} = 3.68 \times 10^3$ kN.m.

Fixed section of the diagonal bracing composite supporting structure (point B to point C) due to effect of the diagonal bracing, the shear graph is a linear change graph. This kind of phenomenon is different from the shear distribution graphics of reinforced anchorage pile. The anchor pile and anti-slide pile the maximum shear force was concentrated on the near of pile bottom. Diagonal bracing composite supporting structure shear maximum value appeared at intersection point of the vertical brace and diagonal bracing.

The shear diagrams of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure are depicted in Fig. 5. Anti-slide pile maximum shear is $Q_{\text{Max}} = 7.61 \times 10^3$ kN. Pre-stressed anchor cable under the effect of pre-stressed anchor cable tension share shear reduced overall structure, maximum shear of $Q_{\text{Max}} = 4.41 \times 10^3$ kN. Original brace structure design calculation has maximum shear of $Q_{\text{Max}} = 2.42 \times 10^3$ kN. Optimize the brace structure design calculation to get maximum shear of $Q_{\text{Max}} = 2.20 \times 10^3$ kN.

Figure 6 shows the displacement diagram of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure. A maximum displacement of anti-slide pile is $X_{\text{Max}} = 75.4$ mm. Pre-stressed anchor cable under the

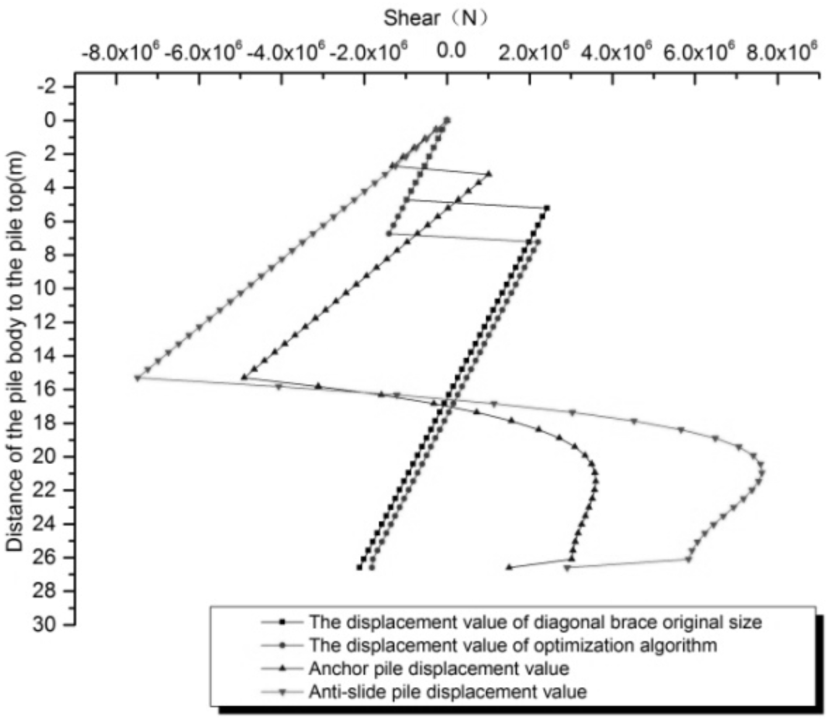

Fig. 5. Shear diagram of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure

effect of pre-stressed anchor cable tension shares less overall structure displacement, the maximum displacement is $X_{\mathrm{Max}} = 33.9\,\mathrm{mm}$. The maximal displacement value of the original design diagonal bracing supporting structure is $X_{\mathrm{Max}} = 2.97\,\mathrm{mm}$. The maximal displacement value of the optimized design diagonal bracing supporting structure is $X_{\mathrm{Max}} = 2.76\,\mathrm{mm}$. The displacement value of diagonal bracing supporting structure relative to anti-slide pile is about 4 %, the displacement value of diagonal bracing supporting structure relative to pre-stressed anchor cable anti-slide pile is about 8.7 %.

4. Results

Three-dimensional finite element analysis base on the practical project diagonal bracing composite supporting structure. The vertical support structure section size $3.0\,\mathrm{m}\times 2.0\,\mathrm{m}$, diagonal support structure section size $2.0\,\mathrm{m}\times 2.0\,\mathrm{m}$, angle between diagonal structure and horizontal is 52° $h_1 = 2700\,\mathrm{mm}$, $h_2 = 12563\,\mathrm{mm}$, $h_3 = 11298\,\mathrm{mm}$ (details of three-dimensional finite element analysis of the entity body are depicted in Fig. 7).

According to the K.J.Bath theory of contact model between different materials,

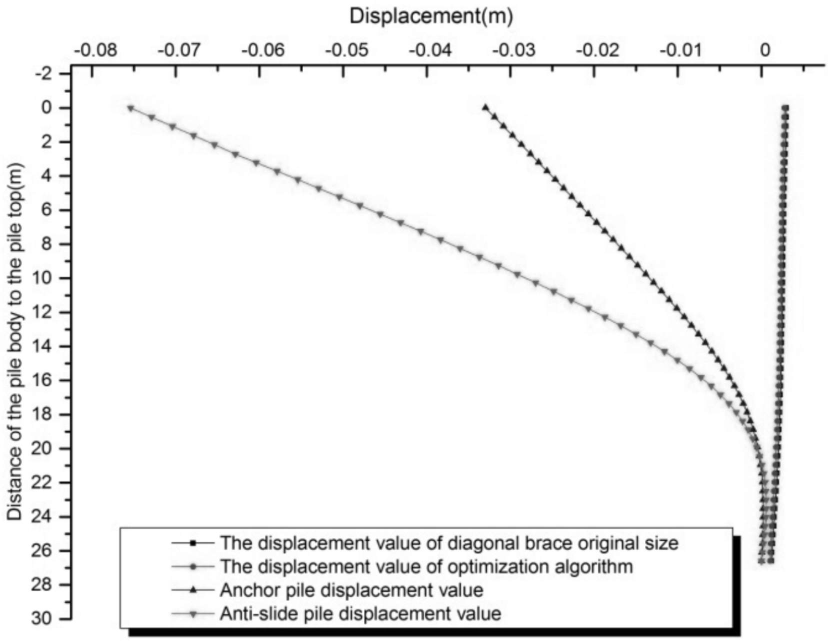

Fig. 6. Displacement diagram of anti-slide pile, reinforced anchorage pile and diagonal bracing composite supporting structure

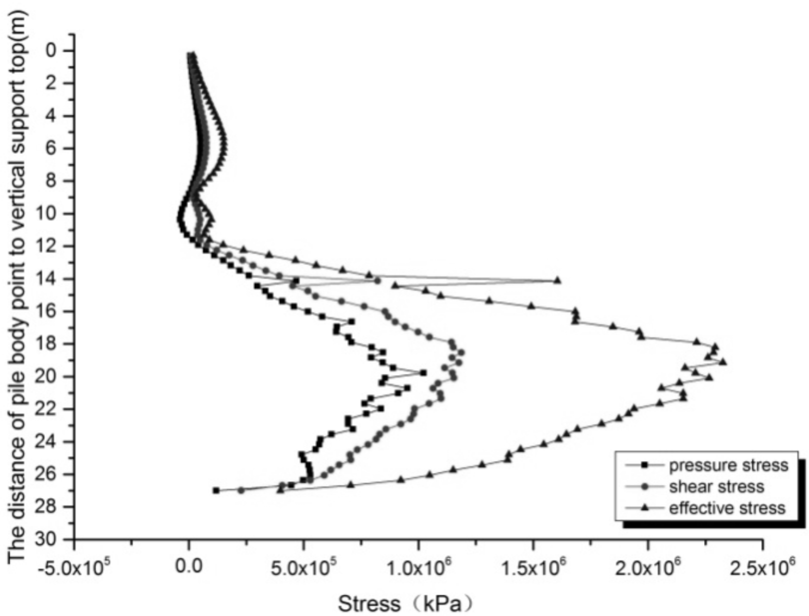

Fig. 7. Vertical brace back lateral principal stress/effective stress analysis diagram

vertical brace is obtained by numerical simulation to solve dorsal soil lateral principal stress analysis diagram (Fig. 8).

Known from the analysis diagram, although the cantilever segments with vertical compressive stress values are increase at intersecting point of vertical brace and diagonal bracing, but compared with the fixed section compressive stress values, cantilever section of the compressive stress value is very small; vertical supporting structure back lateral compressive stress since fixed point to the bottom of the pile increases gradually, near the middle of the fixed section grow to peak. Comparative analysis of the dorsal lateral compressive stress graphics and vertical brace back lateral stress diagram (Fig. 9), back lateral of diagonal bracing composite supporting structure is controlled by compressive stress, dorsal lateral is controlled by tensile stress, the compressive and tensile stresses appeared the corresponding relationship each other; both tensile stress and compressive stress sudden increase at intersecting point of vertical brace and diagonal bracing.

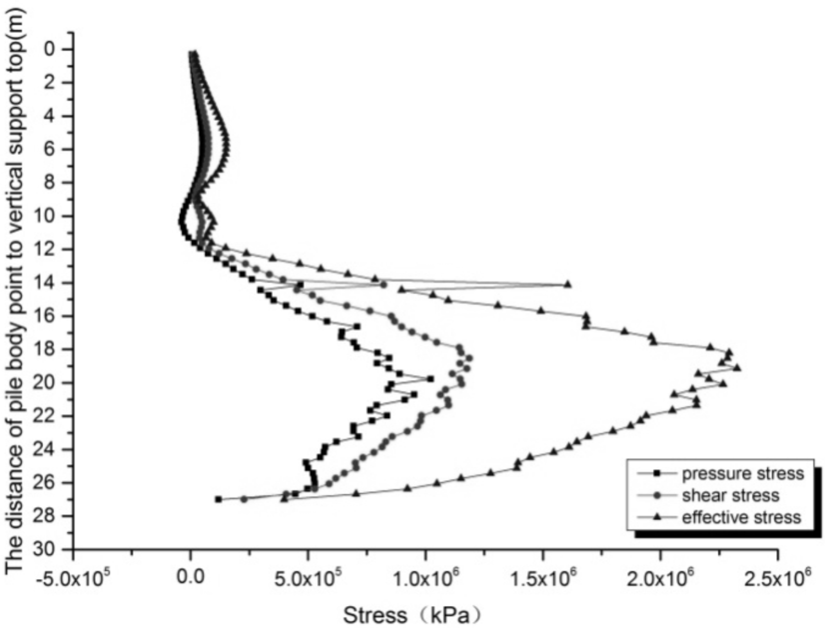

Fig. 8. Vertical brace back lateral compressive stress/shear stress analysis diagram

According to the vertical back lateral principal stress analysis diagram (Fig. 10), back lateral compressive stress larger value corresponds with shear stress of larger value.

Three-dimensional finite element analysis shows that the practical structure as influenced by the geometry size and diagonal bracing supporting foundation, theoretical calculation of the bending moment cannot possible appears zero value, it also does not exist the point in the structure has the zero bending moment, and only affected by the axial force. But through the flexural differential equation algorithm of continuous structure optimized structure can be found in bending moment minimum

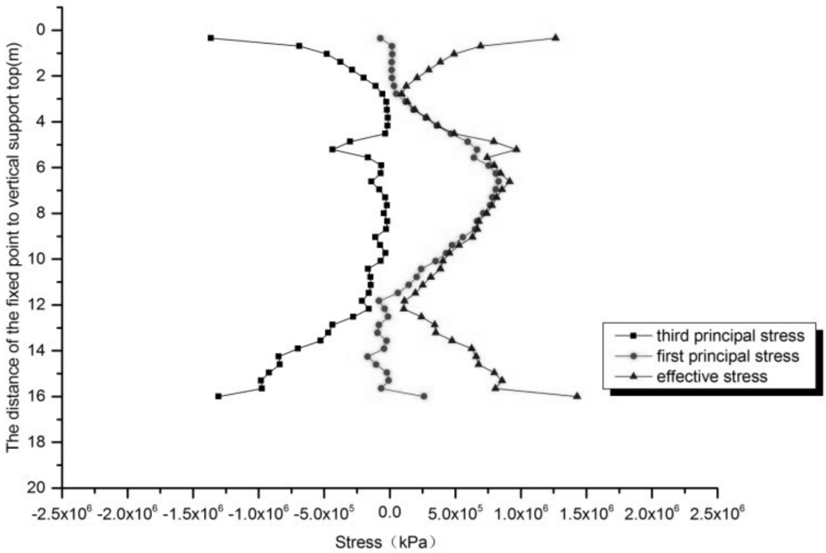

Fig. 9. Vertical brace dorsal lateral principal stress analysis diagram

and axial force of the largest point, so that the concrete compression characteristics of diagonal bracing could be full play, vertical brace of landslide thrust could be fully delivery.

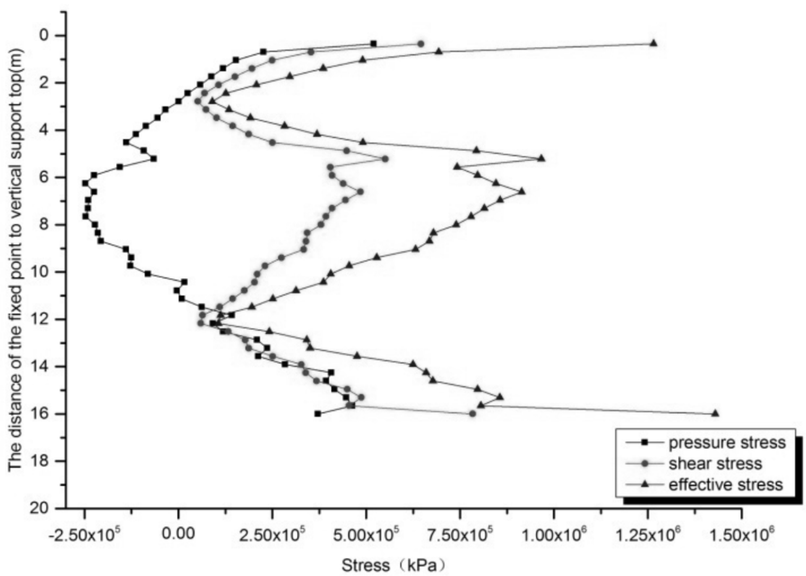

Fig. 10. Vertical brace dorsal lateral compressive stress/shear stress analysis diagram

5. Conclusion

The experimental part analyzes and calculates the diagonal bracing composite supporting structure by using flexural differential equation algorithm of continuous structure. After the experimental analysis, the mechanical characteristic of diagonal bracing composite supporting structure under the landslide thrust are obtained. According to the results for comparing the three-dimensional finite element model and the practical engineering of diagonal bracing composite supporting structure, the following conclusions can be summarized: First, diagonal bracing composite supporting structure with the common existing anti-slide pile supporting structure on the mechanical model has essential difference. Diagonal bracing composite supporting structure is a typical supporting structure is given priority to transfer landslide thrust. Second, through the analysis of the existing structure calculation method, the algorithm is proposed based on the deflection equation of continuous structure. Control conditions are put forward on the brace composite structure. A flexural differential equation algorithm of continuous structure is used to optimize for practical engineering calculation. Base on the above calculation results, optimization design with the original design of the calculated results are compared. At the same time, the original design and optimization design calculation results also will compare with the anti-slide pile and anchor pile calculation results. Finally, by comparison with the results, the internal force distribution characteristics of the diagonal bracing composite supporting structure and advantages are obtained.

Through the above points, it is theoretically verified that the algorithm of continuous structure based on the flexural differential equation is reliability. Meanwhile, optimization calculation for the practical engineering project also achieved good effect. Above all, diagonal bracing composite supporting structure is a kind of effective supporting structure of giant, large landslide control.

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